Volume of solids of Revolution.

About X-axis.

$$V_x = \int_a^b \pi y^2 \, dx$$

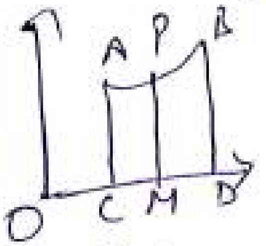
$$V_x = \int_a^b \pi y^2 \frac{dx}{dt} . dt$$

About Y-axis.

$$V_y = \int_a^b \pi x^2 \, dy$$

$$V_y = \int_a^b \pi x^2 \frac{dy}{dt} . dt$$

About any axis.



$$V = \int_{OC}^{OD} \pi (PM)^2 d(OM)$$

$\theta = 0$ initial line

$\theta = \pi/2$ $\perp$er line

About $\theta = 0$

$$V_\theta = \int_\alpha^\beta \frac{2}{3} \pi r^3 \sin\theta \, d\theta$$

$\theta = \pi/2$

$$V_{\theta = \frac{\pi}{2}} = \int_\alpha^\beta \frac{2}{3} \pi r^3 \cos\theta \, d\theta$$

$$\int_\alpha^\beta \frac{2}{3} \pi r^3 \sin(X'OP) \, d\theta$$

P any part

$X'O$ - axis.

Surface of Solids of Revolution

About X-axis

$$V_x = \int_a^b 2\pi y \frac{ds}{dx} . dx$$

$$\frac{ds}{dx} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

$$\int_a^b 2\pi y \frac{ds}{dt} . dt$$

$$\frac{ds}{dt} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$$

$$\int_\alpha^\beta 2\pi y \frac{ds}{d\theta} . d\theta$$

$$\frac{ds}{d\theta} = \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}$$

About Y-axis.

$$V_y = \int_a^b 2\pi x \frac{ds}{dy} . dy$$

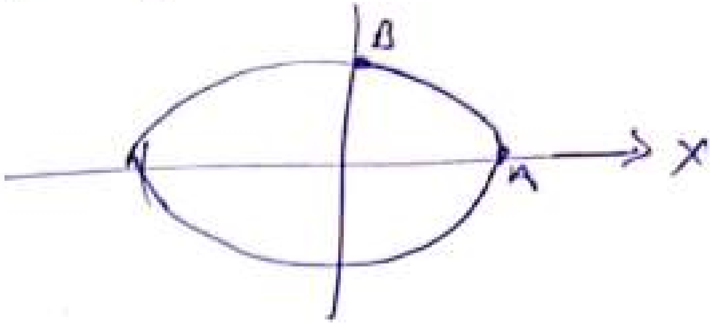
$$\int_a^\beta 2\pi x \frac{ds}{dt} . dt$$

$$\int_\alpha^\beta 2\pi x \frac{ds}{d\theta} . d\theta.$$

.. Find the volume of the Solid generated by revolving the ellips $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ about x-axis.

Solⁿ.

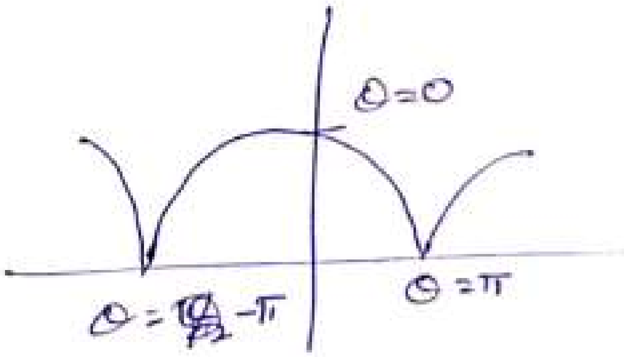
$$V = 2\int_0^a \pi y^2 dx$$



$$\Rightarrow 2\int_0^a \pi b^2\left(1-\frac{x^2}{a^2}\right) dx = \frac{4\pi ab^2}{3}.$$

Q.2. Find the volume of the Solid obtained by revolving one arc of the cycloid $x = a(\theta+\sin\theta)$ $y = a(1+\cos\theta)$ about x-axis.

$$V = 2\int_0^\pi \pi y^2 \frac{dx}{dt}.dt$$



$$\Rightarrow 2\int_0^\pi \pi a^2(1+\cos\theta)^2 . a(1+\cos\theta) d\theta$$

$$\Rightarrow 2\pi a^3\int_0^\pi (1+\cos\theta)^3 d\theta \Rightarrow 5\pi^2 a^3.$$

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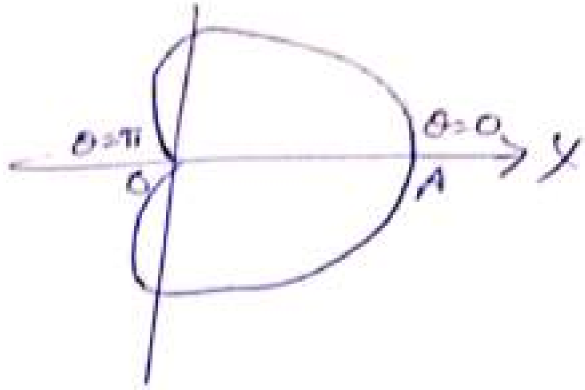
: The curve $r = a(1+\cos\theta)$ revolves about the initial line. Find the volume of the figure formed.

Sol^n: $\Rightarrow$ $r = a(1+\cos\theta)$



$$\int_0^{\pi} \frac{2}{3}\pi r^3 \sin\theta\, d\theta$$

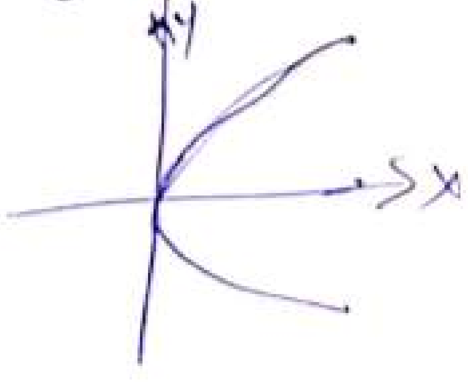
$$= \frac{2}{3}\pi \int_0^{\pi} a^3 (1+\cos\theta)^3 \sin\theta\, d\theta$$

$$= -\frac{2}{3}\pi\, a^3 \int_0^{\pi} (1+\cos\theta)^3 (-\sin\theta)\, d\theta$$

$$\Rightarrow -\frac{2}{3}\pi a^3 \left[\frac{(1+\cos\theta)^4}{4}\right]_0^{\pi} = -\frac{1}{6}\pi a^3 [0-16] = \frac{8}{3}\pi a^3$$

Surface area of the Solids of Revolution.

Q.1. Find the area of the surface formed by the revolution of $y^2 = 4ax$ about the x-axis from vertex to one end of latus rectum.



Sol^n $y = 2\sqrt{a}\sqrt{x}$

$$\frac{dy}{dx} = 2\sqrt{a}\,\frac{1}{2\sqrt{x}} = \frac{\sqrt{a}}{\sqrt{x}}$$

$$\therefore \frac{ds}{dx} = \sqrt{1+\left(\frac{dy}{dx}\right)^2} = \sqrt{1+\frac{a}{x}}$$

$$= \int_0^a 2\pi y \frac{ds}{dx} \cdot dx = 2\pi \int 2\sqrt{a}\cancel{\sqrt{x}} \frac{\sqrt{x+a}}{\cancel{\sqrt{x}}} \, dx$$

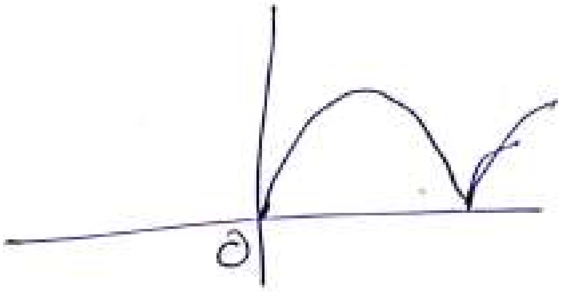
$$\Rightarrow \quad 4\pi\sqrt{a} \int_0^a (x+a)^{1/2} \, dx = 4\pi\sqrt{a} \left[\frac{(x+a)^{3/2}}{3/2} \right]_0^a$$

$$\Rightarrow \quad \frac{8}{3} \pi\sqrt{a} \left[(x+a)^{3/2} \right]_0^a$$

$$= \frac{8}{3} \pi a^2 \left(2\sqrt{2} - 1 \right) \quad \underline{\text{Ans}}.$$

Q. Find the surface area of the solid generated by revolving once complete arch of the cycloid $x = a(\theta - \sin\theta)$ $y = a(1-\cos\theta)$ about x-axis.

Sol^n $\quad x = a(\theta - \sin\theta) \qquad y = a(1-\cos\theta)$



$$\frac{dx}{d\theta} = a(1-\cos\theta)$$

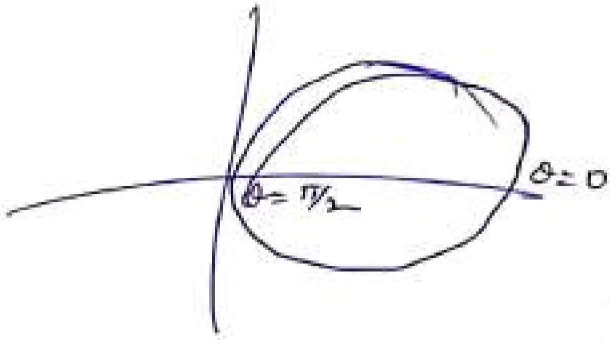
$$\frac{dy}{d\theta} = a\sin\theta$$

$$\frac{ds}{d\theta} = \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2} = 2a\sin\frac{\theta}{2}$$

$$= \int_0^{2\pi} 2\pi y \frac{ds}{d\theta} \, d\theta = 2\pi \int_0^{2\pi} a(1-\cos\theta) \, 2a\sin\frac{\theta}{2} \, d\theta$$

$$\Rightarrow \frac{64\pi a^2}{3} \quad \underline{\text{Ans}}$$

Find the area of the surface of the revolution formed by revolving the curve $r = 2a\cos\theta$ about the initial line.



Soln $\quad r = 2a\cos\theta$

$$\frac{dr}{d\theta} = -2a\sin\theta$$

$$\frac{ds}{d\theta} = \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}$$

$$\Rightarrow \sqrt{(2a\cos\theta)^2 + (-2a\sin\theta)^2} \Rightarrow 2a.$$

$$\Rightarrow \int_0^{\pi/2} 2\pi y \frac{ds}{d\theta} . d\theta \Rightarrow 2\pi \int_0^{\pi/2} r\sin\theta . \frac{ds}{d\theta} . d\theta$$

$$\Rightarrow 2\pi \int_0^{\pi/2} 2a\cos\theta . \sin\theta . 2a . d\theta \Rightarrow 4\pi a^2 \int_0^{\pi/2} \sin 2\theta\, d\theta$$

$$\Rightarrow 4\pi a^2 \left[-\frac{1}{2}\cos 2\theta\right]_0^{\pi/2} = 2\pi a^2 [1+1] = \underline{4\pi a^2}$$